

On Domain-wall/QFT dualities in various dimensions

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Abstract

We investigate domain-wall/quantum field theory correspondences in various dimensions. We give particular emphasis to the special case of the quantum mechanics of 0-branes.

Introduction

Anti-de Sitter (AdS) gravity has attracted much attention due to the conjectured correspondence to a conformal field theory (CFT) on the boundary of the AdS spacetime [1] leading to the so-called AdS/CFT correspondence. (for a review, see [2]). The AdS/CFT correspondence has been extended to a DW/QFT correspondence [3, 4] for Dp -branes in ten dimensions. In this talk we extend the discussion of [3, 4] to general two-block p -branes in various dimensions.

In Section 1 we first discuss some general facts on Domain-Wall and anti-de Sitter spacetimes. In Section 2 we calculate the near-horizon geometries of a generic p -brane. In Section 3 we discuss the field theory limit for the general case. The formulae we give in this Section are applied to the special case of 0-branes we discuss in the Section 4. In this final Section we discuss the quantum mechanics of 0-branes, or extreme black holes, in various dimensions.

1 Domain-Walls and anti-de Sitter spacetimes

Domain-wall (DW) spaces, i.e. spaces of co-dimension 1, occur as solutions to the equations of motion of a (super-)gravity action with a dilaton scalar ϕ and a (D-1)-form gauge potential (for a review, see [5]). They are p -branes with worldvolume dimension $p + 1$ which is one less than the dimension D of their target spacetime, i.e. $D = p + 2$. The part of the supergravity action needed to describe the DW solution is given by (we use the Einstein frame and mostly plus signature):

$$S(D, b) = \int d^D x \frac{\sqrt{-g}}{2\kappa_D^2} \left[-R - \frac{4}{D-2}(\partial\phi)^2 - \frac{1}{2D!} g_s^{2k} \left(\frac{e^\phi}{g_s} \right)^b F_D^2 \right], \quad (1)$$

where κ_D is the gravitational coupling constant with (ignoring constants)

$$\kappa_D^2 \sim \ell_s^{D-2} g_s^2, \quad (2)$$

and b is the dilaton coupling parameter. We also introduced a parameter k defined by

$$k(D, b) = \frac{b}{2} + 2 \frac{D-1}{D-2}. \quad (3)$$

Solving the equations of motions following from (1) one finds:

$$\begin{aligned} ds^2 &= H^{-\frac{4\varepsilon}{(D-2)\Delta_{\text{DW}}}} dx_{D-1}^2 + H^{\frac{-4\varepsilon(D-1)}{(D-2)\Delta_{\text{DW}}} - 2(\varepsilon+1)} dy^2, \\ e^\phi &= g_s H^{\frac{-(D-2)b\varepsilon}{4\Delta_{\text{DW}}}}, \\ g_s^k F_{01\dots D-2y} &= \pm \sqrt{\frac{4}{\Delta_{\text{DW}}}} \partial_y H^\varepsilon, \end{aligned} \quad (4)$$

with ε a parameter and where g_s and Δ_{DW} (which is invariant under dimensional reduction) are defined by

$$\begin{aligned} g_s &= e^{\phi(H=1)}, \\ \Delta_{\text{DW}}(D, b) &= \frac{1}{8}(D-2)b^2 - 2 \frac{D-1}{D-2}. \end{aligned} \quad (5)$$

The function H is harmonic on the 1-dimensional transverse space with coordinate y :

$$\begin{aligned} H &= c + Q_1 y, & y > 0, \\ H &= c + Q_2 y, & y < 0, \end{aligned} \quad (6)$$

with c, Q_1, Q_2 constant. The domain-wall is positioned at the discontinuity $y = 0$. Different choices of ε correspond to different choices of coordinates and lead to different expressions for the metric [6]. For instance, one can choose ε such that the powers of the harmonic function become equal or opposite.

Shifting the position of the domain wall to infinity, e.g. $y \rightarrow +\infty$ allows us to discard the constant c in the harmonic function and, furthermore there is only one side of the domain wall. We can eliminate ε if we define a mass parameter m by :

$$Q\varepsilon = m, \quad (7)$$

with $Q = Q_1$. Making a co-ordinate transformation and by going to the so-called “dual frame” metric g_* , indicated by a subscript star,

$$Qy = e^{-Q\lambda}, \quad g_* = e^{-b\phi} g_E, \quad (8)$$

we find for the solution (4)

$$ds_*^2 = e^{-2m\lambda(\frac{2+\Delta_{\text{DW}}}{\Delta_{\text{DW}}})} dx_{D-1}^2 + d\lambda^2, \quad \phi = \lambda \frac{(D-2)bm}{4\Delta_{\text{DW}}}. \quad (9)$$

In the dual frame (8) the domain-wall solution (9) describes an AdS_D spacetime with a linear dilaton, the latter breaking the full conformal structure of the AdS spacetime. This observation will be the key to generalizing the AdS/CFT duality to a DW/QFT duality.

2 Near-horizon Geometries of p -branes

Our starting point is the D -dimensional action

$$S(D, a, p) = \int d^D x \frac{\sqrt{-g}}{2\kappa_D^2} \left[-R - \frac{4}{D-2} (\partial\phi)^2 - \frac{g_s^{2k}}{2(d+1)!} \left(\frac{e^\phi}{g_s} \right)^a F_{d+1}^2 \right], \quad (10)$$

which contains three independent parameters: the target spacetime dimension D , the dilaton coupling parameter a and a parameter p specifying the rank $D - p - 2$ of the field strength F . The parameter k is a generalization of (3) and is given by

$$k(D, a, p) = \frac{a}{2} + 2 \frac{p+1}{D-2}. \quad (11)$$

We have furthermore introduced two useful dependent parameters d and \tilde{d} which are defined by

$$\begin{cases} d &= p+1 & \text{dimension of the worldvolume,} \\ \tilde{d} &= D-p-3 & \text{dimension of the dual brane worldvolume.} \end{cases} \quad (12)$$

We next consider the following class of diagonal “two-block” p -brane solutions (using the Einstein frame)¹:

$$\begin{aligned} ds^2 &= H^{-\frac{4\tilde{d}}{(D-2)\Delta}} dx_d^2 + H^{\frac{4d}{(D-2)\Delta}} dx_{\tilde{d}+2}^2, \\ e^\phi &= g_s H^{\frac{(D-2)a}{4\Delta}}, \\ g_s^{(2-k)} \tilde{F}_{m_1 \dots m_{\tilde{d}+1}} &= \pm \sqrt{\frac{4}{\Delta}} \epsilon_{m_1 \dots m_{\tilde{d}+1} m} \partial_m H, \end{aligned} \quad (13)$$

where

$$g_s^{(2-k)} \tilde{F} = \left(\frac{e^\phi}{g_s} \right)^a g_s^k {}^* F. \quad (14)$$

We use a constant, i.e. metric independent, Levi-Civita tensor. Furthermore, $g_s = e^{\phi(H=1)}$ and Δ is a generalization of the Δ_{DW} in the previous section defined by [7]

$$\Delta = \frac{1}{8}(D-2)a^2 + \frac{2d\tilde{d}}{D-2}, \quad (15)$$

which is invariant under reductions and oxidations (in the Einstein frame). The function H is harmonic over the $\tilde{d} + 2$ transverse coordinates and, assuming that

$$\tilde{d} \neq -2, 0, \quad (16)$$

(i.e. no constant or logarithmic harmonic) this harmonic function is given by

$$H = 1 + \left(\frac{r_0}{r} \right)^{\tilde{d}}. \quad (17)$$

Here r_0 is an integration constant with the dimensions of length. It is related to the mass and charge of the p -brane as follows. The mass τ_p per unit p -volume is given by the ADM-formula:

$$\begin{aligned} \tau_p &= \frac{1}{2\kappa_D^2} \int_{\partial M^{D-p-1}} d^{D-p-2} \Sigma^m (\partial^n h_{mn} - \partial_m h^b_b) \\ &= \frac{2(D-p-3)}{\Delta \kappa_D^2} r_0^{D-p-3} \Omega_{D-p-2}. \end{aligned} \quad (18)$$

On the other hand, the charge μ_p per unit p -volume is given, in terms of the same integration constant r_0 , by the Gauss-law formula

$$\mu_p = \frac{1}{2\kappa_D^2} \int (d^{D-p-2} \Sigma)^{m_1 \dots m_{D-p-2}} g_s^{(2-k)} \tilde{F}_{m_1 \dots m_{D-p-2}}$$

¹For later convenience, we give the solution in terms of the *magnetic* potential of rank $D-p-3$. The p -brane solution is *electrically* charged with respect to the $p+1$ -form potential.

$$= \pm \sqrt{\frac{\Delta}{4}} \tau_p. \quad (19)$$

Hence, the p-brane solution satisfies the BPS bound

$$\tau_p = \sqrt{\frac{4}{\Delta}} |\mu_p|. \quad (20)$$

To derive an expression for r_0 in terms of the string parameters g_s and ℓ_s , which fixes the scaling of H , one must add a source term to the supergravity bulk action. Using the no-force condition² and the fact that in the string frame the electric (p+1)-form potential C_{p+1} is proportional to g_s^{-k} , as follows from the action (10), we find that

$$\tau_p \sim \frac{1}{\ell_s^{p+1} g_s^k}. \quad (21)$$

Comparing with (18) and using (2) we deduce that, for a single brane,

$$\left(\frac{r_0}{\ell_s}\right)^{\tilde{d}} \sim g_s^{2-k}. \quad (22)$$

The two-block solutions (13) include the (supersymmetric) domain-wall spaces of the previous section. They correspond to the case $\tilde{d} = -1$, $\varepsilon = -1$ and $r_0 = 1/m$. The solutions also include the known branes in ten and eleven dimensions (M2, M5, Dp, F1, NS5 etc.) as well as branes in lower dimensions. If the branes under consideration preserve any supersymmetries we can set [7]

$$\Delta = \frac{4}{n}, \quad (23)$$

where generically $32/2^n$ is the number of unbroken supersymmetries.

We now consider the limit for which the constant part in the harmonic function is negligible. As in the previous section we make a co-ordinate transformation and go to the dual frame

$$\left(\frac{r_0}{r}\right) = e^{-\lambda/r_0} \quad g_* = e^{(\frac{a}{d})\phi} g_E. \quad (24)$$

After these manipulations we can write the near-horizon metric as

$$ds_*^2 = e^{-2(1-\frac{2\tilde{d}}{\Delta})\lambda/r_0} dx_d^2 + d\lambda^2 + r_0^2 d\Omega_{d+1}^2, \quad \phi = -\lambda \frac{(D-2)a\tilde{d}}{4\Delta r_0}, \quad (25)$$

²Alternatively, one can use a scaling argument, see Appendix B of [8].

which has an $AdS_{d+1} \otimes S^{\tilde{d}+1}$ geometry and a linear dilaton.

Reducing over the $\tilde{d} + 1$ angular variables of the sphere we end up with a gauged supergravity in $d + 1$ dimensions of the form (1) supporting a domain-wall solution of the form (4). The precise relation between the parameters of the action (1) and its solution (4) in terms of those of the action (10) and its solution (25) can be found in section 2.3 of [8].

Summarizing, in this Section we showed that in the dual frame, defined by (24), all p -branes solutions (13) have a near horizon geometry $DW_{d+1} \otimes S^{\tilde{d}+1}$. The domain-wall metric has all the isometries of an AdS space. These isometries are broken in the full background because of the presence of a linear dilaton.

3 The field theory limit

In this section we will set up the framework for the DW/QFT duality similar to the analysis of [4] but for arbitrary dimensions. The analysis is similar to the AdS/CFT duality. There one finds (in the low-energy limit) that supergravity in the near-horizon geometry of a large number of non-dilatonic branes is dual to the conformal field theory living on these branes (which are located at the boundary of the near-horizon geometry). This so-called holographic principle lies at the heart of the AdS/CFT conjecture.

As we showed in the previous sections, the near-horizon geometry of a general p -brane in the dual-frame is equivalent to that of a non-dilatonic p -brane. It is therefore natural to assume that the duality might be extended. The presence of the dilaton makes the AdS background into a domain-wall background and the conformal field theory into a general quantum field theory, hence the name DW/QFT duality. In the following we will only consider D-branes and their intersections in lower dimensions.

One starts with a Dp -brane configuration in string theory and takes the low-energy limit in which only the massless modes survive and in which the theories in the bulk and on the brane decouple. Next, one introduces a new energy-scale and a dimensionless coupling constant which are the free parameters of the decoupled theories. The energy-scale can be given in several ways: If one probes the stacked branes by another Dp -brane, a natural energy-scale is given by the mass of the endpoint of a stretched string, which acts like a W -boson (see Figure 1)

$$E_W = U = \frac{r}{l_s^2}. \quad (26)$$

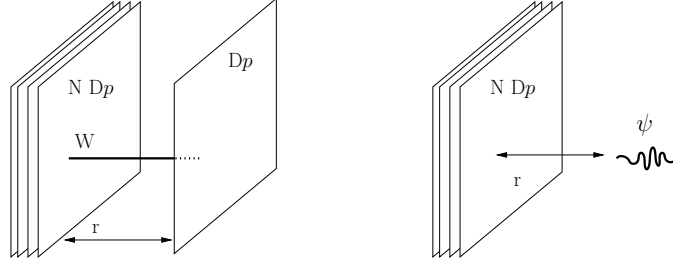


Figure 1: Probing a stack of N Dp -branes can either be done by another Dp -brane (left) or by a supergravity field ψ (right)

This was the approach taken by [3], all branes have the same energy-scale but the disadvantage is that the near-horizon geometry when written in the U co-ordinates does not take the same form for all branes. One can also probe the branes by a supergravity field ψ (suppressing possible quantum numbers). By solving the wave equation for this field [9] one finds the following energy scale

$$E_\psi = u = \frac{r^\beta}{r_0^{\beta+1}}, \quad \beta = \frac{2\tilde{d}}{\Delta} - 1, \quad r_0 = l_s (g_s^{2-k} N)^{\frac{1}{\tilde{d}}}. \quad (27)$$

In [10] it was shown that this energy-scale corresponds to the so-called holographic energy-scale used in entropy calculations. Although the energy depends on the parameters of the brane solution, one finds that the near-horizon geometry (9) in these holographic u co-ordinates is particularly simple:

$$ds_*^2 = r_0^2 \left[\mathcal{R}^2 \left\{ u^2 dx_d^2 + \left(\frac{du}{u} \right)^2 \right\} + d\Omega_{\tilde{d}+1}^2 \right]. \quad (28)$$

For all branes this metric has the form $AdS(\mathcal{R}r_0)_{d+1} \otimes S^{\tilde{d}+1}(r_0)$ differing only in the (relative) radii of the sphere and anti-de-Sitter space.

The quantum field theory on the brane is in general not explicitly known, but dimensional analysis enables us to extract some information. If we assume that the theory is given by some q -form potential on the $d = p + 1$ dimensional worldvolume

$$S_{wv} = \tau_p \int d^{p+1} \xi \text{Tr} F_{q+1}^2, \quad (29)$$

then we have for the coupling constant scaling behavior

$$g_{ft} \sim g_s^k l_s^\alpha \quad \text{with} \quad \alpha = p - 2q - 1. \quad (30)$$

For 10-dimensional Dp -branes we have a vector-multiplet (i.e. $q = 1$) on the worldvolume reproducing the familiar $\alpha = p - 3$ scaling behavior for the Yang-Mills theory on the Dp -brane worldvolume. The effective dimensionless coupling constant λ is given by

$$\lambda_W = g_{ft} N E_W^\alpha, \quad \lambda_\psi = g_{ft} N E_\psi^\alpha. \quad (31)$$

Since we have two different probes, we can in principle construct two different dimensionless couplings, constructed by either E_W or E_ψ . Since both probes have a sensible interpretation, it must be possible to use them both, independently of the low-energy limit. This leads to the following constraint on the scaling behavior in terms of the brane-solution parameters [8]:

$$\alpha = \Delta - \tilde{d} = a \frac{D - 2}{4}. \quad (32)$$

Combining this with the definitions of the probe-scales E_W and E_ψ we find the following relations between them and their corresponding dimensionless couplings λ_W and λ_ψ

$$\frac{E_W}{E_\psi} = \lambda_W^{\frac{2}{\Delta}}, \quad \lambda_W^\beta = \lambda_\psi. \quad (33)$$

The decoupling limit is now given by

$$\begin{array}{ll} \text{take} & E/E_s \rightarrow 0, \quad E_s = \ell_s^{-1}, \\ \text{at fixed} & \begin{cases} \lambda_W = g_{ft} N E_W^\alpha & \text{and} & E/E_W \\ \lambda_\psi = g_{ft} N E_\psi^\alpha & \text{and} & E/E_\psi \end{cases} \quad \text{or, equivalently, see (33),} \end{array}$$

The dimensionless coupling made from Newton's constant in this limit becomes

$$G_N = g_s^2 \left(\frac{E}{E_s} \right)^{D-2} = \left(\frac{\lambda}{N} \right)^2 \left(\frac{E}{E_s} \right)^{D-2-2\alpha} \quad (34)$$

so that if α becomes too large one has to take N to infinity to ensure the decoupling of gravity.

In the AdS/CFT correspondence one has in the field theory side the parameters $(\lambda, \frac{1}{N})$ which correspond to the curvature and string coupling on the supergravity side. They generalize to the effective tension of a string in the dual frame and the dilaton, respectively:

$$\tau_* l_s^2 = \lambda_\psi^{\frac{2}{\Delta}}, \quad e^\phi = \frac{\tau_*^{\frac{\tilde{d}}{2}}}{N}. \quad (35)$$

We now have the following ranges of validity on the two sides of the duality

- perturbative SYM:

$$\lambda_W \ll 1 \Rightarrow \begin{cases} \alpha < 0 : E_W \rightarrow \infty : & \text{UV-free,} \\ \alpha > 0 : E_W \rightarrow 0 : & \text{IR-free.} \end{cases}$$

- classical SUGRA:

$$\begin{cases} \tau_* l_s^2 \gg 1 : & \text{no stringy corrections,} \\ e^\phi \ll 1 : & \text{no quantum corrections.} \end{cases}$$

Using the above formulae one can easily see that classical supergravity describes strongly coupled, large N field theory. However, the conformal invariance which in the AdS/CFT duality facilitates computations in the strongly coupled field theory is now broken so that any direct check of a DW/QFT duality is ruled out. For specific examples, we refer to [8].

As long as β is positive, the supergravity and field theory are valid in different regimes (i.e. $\lambda_\psi \gg 1$ and $\lambda_W \ll 1$) and one can find consistent phase diagrams as in [3]. However, for negative β , e.g. in case of the D6-brane, this seems no longer true at first sight: $\lambda_\psi \ll 1$ and $\lambda_W \ll 1$.

As one can see by using the relation (33) between the two couplings, this corresponds to $\lambda_W \gg 1$ and $\lambda_W \ll 1$. This matches nicely with the explanation of [9], namely that the low-energy Hilbert space of the field theory has two separate sectors: one describing nearby (in U co-ordinates) brane probes and one describing supergravity probes far away (also in U co-ordinates) from the brane.

Finally we note that when the dilaton becomes larger one can perform an S-duality transformation. If the curvature is small with respect to the S-dual string scale, then one can still trust the supergravity approximation.

4 Dynamics of 0-branes

In this Section we consider the special case of 0-branes in various dimensions. The (bosonic) Lagrangian for a particle with mass m and charge q , moving in the string frame near-horizon background of N stacked 0-branes reads

$$\mathcal{L} = m e^{-\phi} \sqrt{|\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}^S|} + q A_\mu \dot{x}^\mu, \quad (36)$$

where the dot represents derivatives with respect to the worldline time. The D-dimensional 0-brane solution in the dual frame $g_{\mu\nu}^*$ is given by the expression

(28), taken for $p = 0$. Introducing the canonical momentum $P_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$ one can write down the mass-shell constraint in the string frame for the probe particle as

$$(P_\mu - qA_\mu)(P_\nu - qA_\nu)g_S^{\mu\nu} = m^2 e^{-2\phi}. \quad (37)$$

We would like to solve this equation for $P_t = -\mathcal{H}$. For this purpose, we transform the mass-shell equation (37) to the dual frame and substitute the metric and gauge field of the solution (28). For the gauge field (being in the temporal gauge) we can write:

$$\frac{A_t}{u} = r_0 e^{-\frac{D-4}{D-3}\phi} \equiv M, \quad (38)$$

and we find for the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \left(\frac{u}{\mathcal{R}}\right)^3 \frac{P_u^2}{2f} + \frac{u}{\mathcal{R}} \frac{g}{2f}, \\ f &= \frac{1}{2} \left(qM + \sqrt{(mM)^2 + \left(\frac{uP_u}{\mathcal{R}}\right)^2 + \vec{L}^2} \right), \\ g &= \vec{L}^2 + (m^2 - q^2)M^2. \end{aligned}$$

We can write this Hamiltonian in a rather suggestive form by making the following transformation

$$\frac{u}{\mathcal{R}} = \frac{(2\mathcal{R})^2 M}{x^2}. \quad (39)$$

after which the Hamiltonian takes on the following form

$$\begin{aligned} \mathcal{H} &= \frac{P_x^2}{2f} + \frac{g}{2fx^2}, \\ f &= \frac{1}{2} \left(q + \sqrt{m^2 + \left(\frac{xP_x}{2\mathcal{R}M}\right)^2 + \left(\frac{\vec{L}}{M}\right)^2} \right), \\ g &= 4 \left(\vec{L}^2 + (m^2 - q^2)M^2 \right). \end{aligned}$$

The dynamics of the system is now generated by the Poisson brackets with respect to the canonical co-ordinate and momentum (x, P_x) . The transition to quantum mechanics is the usual one. Defining two other operators

$$\mathcal{D} = \frac{1}{2}xP_x, \quad \mathcal{K} = -\frac{1}{2}fx^2, \quad (40)$$

we find that the system possesses a classical $SL(2, \mathbb{R})$ -symmetry

$$\{\mathcal{D}, \mathcal{H}\} = \mathcal{H} \quad \{\mathcal{D}, \mathcal{K}\} = -\mathcal{K} \quad \{\mathcal{H}, \mathcal{K}\} = 2\mathcal{D}. \quad (41)$$

This is only true when the effective coupling constant λ_ψ of the model under consideration is fixed under scale transformations. Whenever we have a non-trivial dilaton the model will not be conformally invariant by itself. Only when introducing a transformation to keep λ_ψ fixed under conformal transformations, will the model be invariant under what are called generalized conformal transformations [11].

Taking the limit $Mm \rightarrow \infty$ and at the same time $M(m - q) \rightarrow 0$ [12] we find a “non-relativistic” Hamiltonian of the form:

$$\mathcal{H} = \frac{P_x^2}{2m} + \frac{\vec{L}^2}{2mx^2}. \quad (42)$$

This model, as first considered in [13], is in fact an integrable model so that it might offer a concrete test of the DW_2/QFT_1 duality in this particular limit.

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